PhODS MOCK PHYSICS COMPETITION 2024

2024 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Test under standard conditions, meaning that you must complete the test in 75 minutes in one sitting.
- This test contains 25 multiple-choice questions. **Your answer to each question must be marked on the Google Forms answer sheet that accompanies the test.**
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- All questions are equally weighted but are not necessarily the same level of difficulty.
- Join our discord to discuss the exam and join a large physics community!
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 1, 2024 EST.**

We acknowledge the following people for their contributions to this year's exam (in alphabetical order):

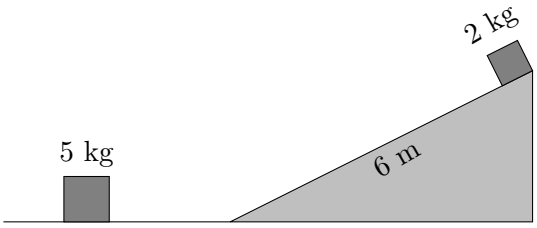
Aarush Deshpande, Accuide, Aarav Kansal, Alan Wang, Richard Yang, and Nathan Zhao

Special thanks to all test solvers who were able to give helpful feedback on the problems.

1. Bob pushes a 3.6 kg textbook with a horizontal force of 48 N on a flat table with coefficients of friction $\mu_s = \mu_k = 0.75$. What is the magnitude of the total force the book exerts on the table?

   (A) 27 N (B) 45 N (C) 48 N (D) 60 N (E) 63 N

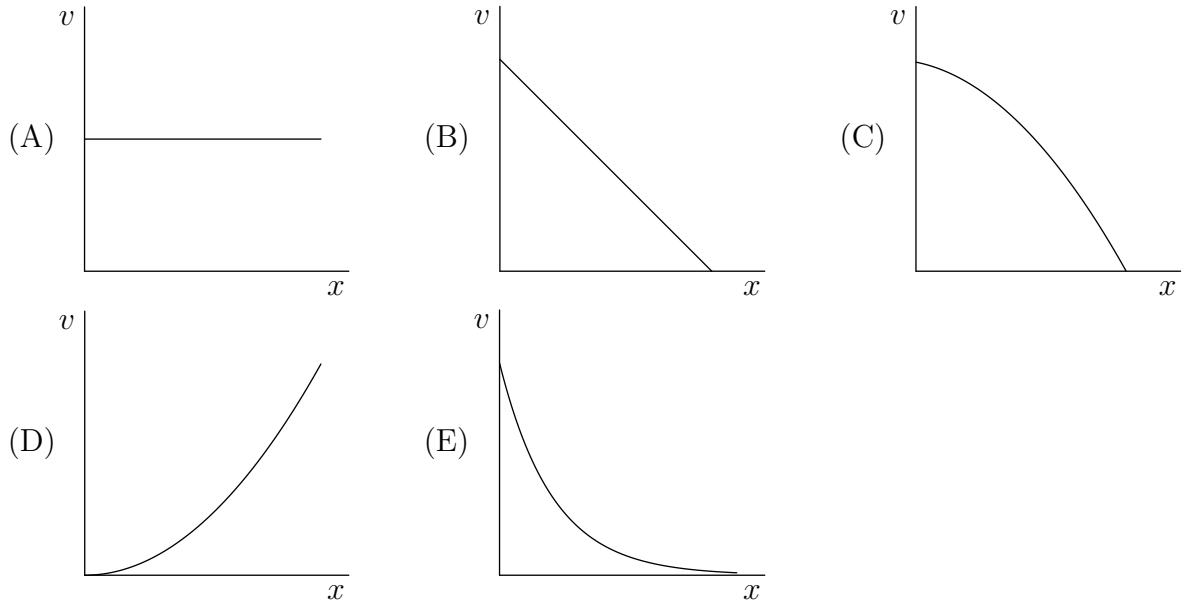
2. A 2 kg block slides smoothly down a 6 m long ramp of angle 30° and collides completely inelastically with a 5 kg mass at the bottom. How much energy is lost to heat? Assume all surfaces are frictionless and the ramp is fixed in place.



Block Collision

   (A) 17.1 J (B) 26.4 J (C) 33.6 J (D) 42.9 J (E) 47.8 J

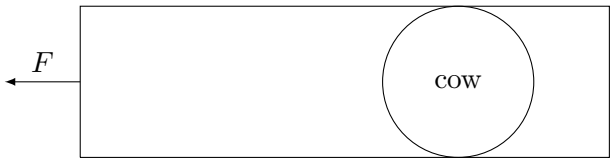
3. Which of the following illustrates the motion of a block sliding on a surface with a constant coefficient of kinetic friction?



4. A car has mass 1216 kg (excluding the wheels) and four wheels of uniform mass density, diameter 50 cm, and mass 16 kg. A constant retarding force of 400 N acts on the car. What is the power required to accelerate the car from rest to 90 km/h in 10 seconds? Assume all wheels roll without slipping and the acceleration is constant.

   (A) 44 kW (B) 45 kW (C) 46 kW (D) 47 kW (E) 51 kW

5. Farmer Nhoj shoves a spherical (approximately) cow with diameter h and constant density ρ into a trailer of height h and mass M. Farmer Nhoj's truck pulls it with an acceleration a. If the cow does not move relative to the trailer, what is the force necessary to pull the trailer?



Cow in Trailer

(A) Ma (B) $\left(M + \frac{\pi\rho h^3}{21}\right)a$ (C) $\left(M + \frac{\pi\rho h^3}{6}\right)a$

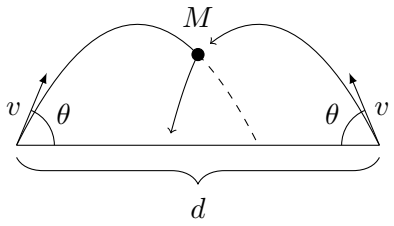
(D) $\left(M + \frac{2\pi\rho h^3}{3}\right)a$ (E) $\left(M + \frac{4\pi\rho h^3}{3}\right)a$

6. Two identical masses are launched towards each other at the exact same time with an initial velocity v and launch angle θ from a distance $d > 0$ apart. They collide partially inelastically midair at point M. Which of the following statements is true? Neglect air resistance.

I. The two masses land at the exact same time

II. The two masses land at the same distance to M

III. The masses land at the same time as they would have landed if they didn't collide



Midair Collision (dashed lines show trajectory without collision)

(A) Only I (B) Only II (C) I and II (D) I and III (E) I, II, and III

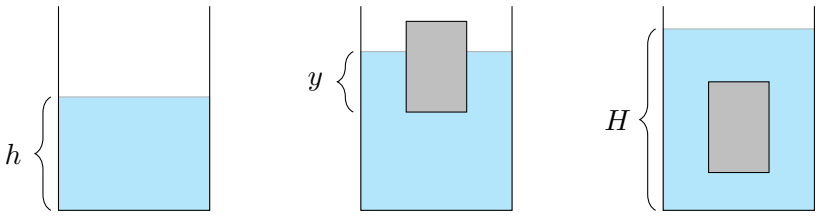
7. Farmer John has created an exercise routine for his N cows conveniently numbered $1...N$. Cow i has mass i^2 kilograms and stands i^3 meters from the center of a massless carousel. The carousel rotates with an angular velocity such that the outermost cow has a speed of N^4 meters per second. The angular momentum of the cows about the center of the carousel is proportional to

(A) N^9 (B) N^{10} (C) N^{11} (D) N^{12} (E) N^{13}

8. Suppose the power generated by a wind turbine depends only on the density ρ of the air, the length L of each turbine blade, the speed v of the wind, and a dimensionless constant k proportional to the turbine's efficiency. Which of the following expresses the turbine's power output?

(A) $k\rho vL$ (B) $k\rho v^2 L$ (C) $k\rho v^2 L^2$ (D) $k\rho v^3 L$ (E) $k\rho v^3 L^2$

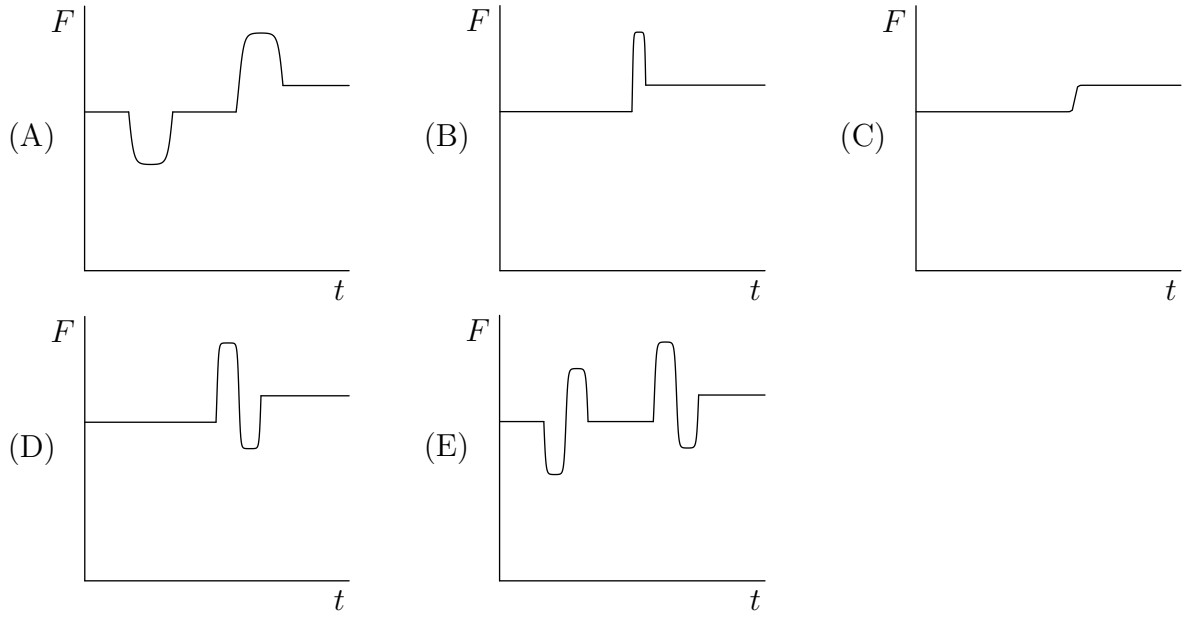
9. A cylindrical vessel of radius R is filled to height h with water of density ρ_w. The water level rises by y meters when a cylindrical object of radius r and density ρ_c floats in it. What is the water level H when the object is submerged?



Drown or Float?

(A) $h + \dfrac{\rho_w}{\rho_c} y$ (B) $h + \dfrac{\rho_c}{\rho_w} y$ (C) $h + \dfrac{\rho_w r^2}{\rho_c R^2} y$ (D) $y + \dfrac{\rho_c}{\rho_w} h$ (E) $y + \dfrac{\rho_w r^2}{\rho_c R^2} h$

10. Elppaenip is standing on a scale when he quickly bends down, picks up a pineapple on the floor, and stands back up. Assuming he does not lean on the pineapple, which graph could represent the weight on the scale over time?



11. A truck of mass M fully loaded with a mass m of pineapples is driving at speed v on a bridge when the back door opens. The pineapples fall out of the truck at a strictly decreasing rate. Which of the following is true? Do NOT neglect air resistance, which acts horizontally on falling pineapples.

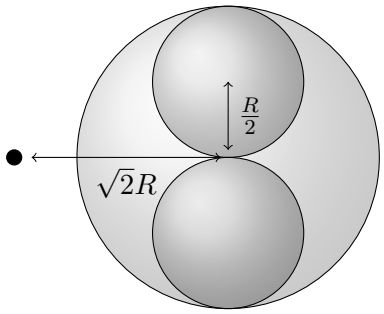
I. The truck and pineapples apply an average vertical force of $(M + m)\, g$ to the bridge

II. The truck and pineapples apply a maximum vertical force of $(M + m)\, g$ to the bridge

III. The pineapples apply a total horizontal impulse of mv to the bridge before coming to rest

(A) Only I (B) Only II (C) Only III (D) I and III (E) None of these

12. Due to its inherent need to be special, the magic planet PhODS-land has two spherical cavities of radius $R/2$ at $y = \pm R/2$. It has a radius R, a mass M after removing the cavities, and uniform density. What is the magnitude of the gravitational field at a distance $\sqrt{2}R$ from the center of PhODS-land?

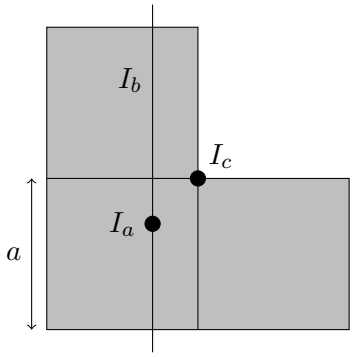


PhODS-land Gravity

(A) $\dfrac{GM}{2R^2}\left(1 - \dfrac{4\sqrt{2}}{27}\right)$ (B) $\dfrac{7GM}{18R^2}$ (C) $\dfrac{GM}{2R^2}$

(D) $\dfrac{14GM}{27R^2}$ (E) $\dfrac{2GM}{3R^2}\left(1 - \dfrac{4\sqrt{2}}{27}\right)$

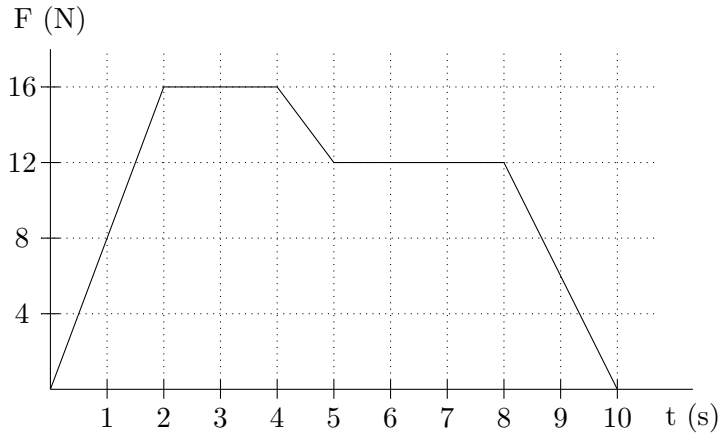
13. Three identical squares with uniform mass distribution are welded together. Let I_a, I_b, and I_c be the moment of inertia of the shape about an axis through its center of mass perpendicular to its plane, center of mass in its plane, and the point where all three squares touch perpendicular to the plane, respectively. Which of the following inequalities is true?



Combined Squares

(A) $I_a < I_b < I_c$ (B) $I_a = I_b < I_c$ (C) $I_b < I_a < I_c$ (D) $I_c < I_a = I_b$ (E) $I_c < I_a < I_b$

14. A particle with mass 4 kg starts at rest and undergoes a force F in the positive x-direction that varies with time as shown in the graph below. Which of the following is true?



(A) The power at $t = 4$ is less than at $t = 8$; The kinetic energy of the particle is maximized at $t = 8$

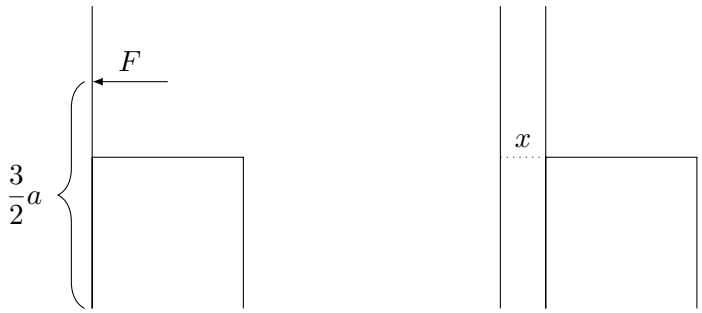
(B) The power at $t = 4$ is less than at $t = 8$; The kinetic energy of the particle is maximized at $t = 10$

(C) The power at $t = 4$ is greater than at $t = 8$; The kinetic energy of the particle is maximized at $t = 4$

(D) The power at $t = 4$ is greater than at $t = 8$; The kinetic energy of the particle is maximized at $t = 8$

(E) The power at $t = 4$ is greater than at $t = 8$; The kinetic energy of the particle is maximized at $t = 10$

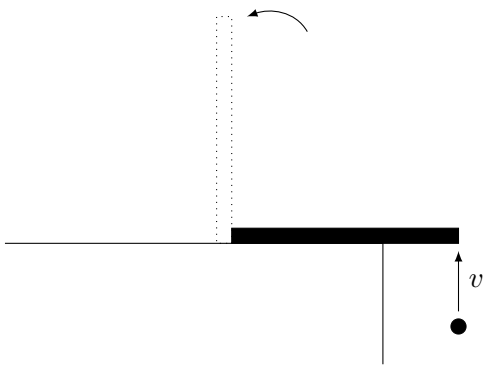
15. Timmy's chair is modeled below as 4 uniform rods of length a and mass m. He barely tips the chair back with force F applied at height $\frac{3}{2}a$. His father adds an anti-tipping rod of length $2a$ and mass $2m$ to the back of his chair at distance x. What is the minimum x such that Timmy cannot tip the modified chair with a force of magnitude F applied anywhere? Assume the chair does not slide.



Original Chair Anti-tip Chair

(A) $\frac{1}{8}a$ (B) $\frac{1}{6}a$ (C) $\frac{1}{4}a$ (D) $\frac{1}{3}a$ (E) $\frac{1}{2}a$

16. A thin slab of mass M and length ℓ hangs over a ledge. A bullet of mass m and velocity v hits the bottom edge of the slab and gets stuck inside it. The slab comes to rest perfectly vertically. Find v. Assume the slab does not slip.

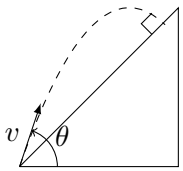


Bullet Hits Slab

(A) $\dfrac{M+m}{m}\sqrt{gl}$ (B) $\sqrt{\dfrac{g\ell(M+2m)(M+m)}{2m^2}}$ (C) $\sqrt{\dfrac{g\ell(M+m)(M+2m)}{3m^2}}$

(D) $\sqrt{\dfrac{g\ell(M+2m)(M+3m)}{3m^2}}$ (E) $\sqrt{\dfrac{2g\ell(M+m)(M+3m)}{3m^2}}$

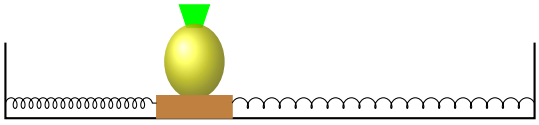
17. A projectile is launched at angle θ to the horizontal from the base of an inclined plane of angle $45°$ and lands with its velocity perpendicular to the plane. Which of the following is closest to θ? Neglect air resistance.



Perpendicular Projectile

(A) 54° (B) 60° (C) 66° (D) 72° (E) 78°

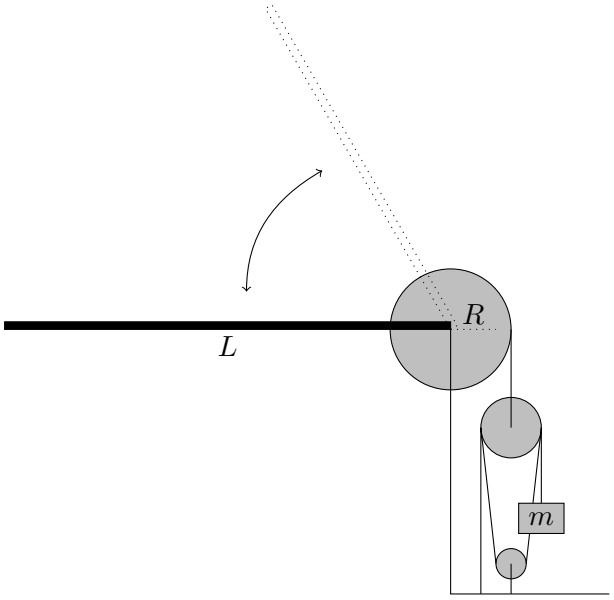
18. Two ideal springs with spring constant 2.1 N/m are attached to a massless cutting board with a pineapple of mass 960 g. A chef holds a double-edged knife, which removes 10% of the remaining mass after each cut instantaneously each time the pineapple passes the equilibrium point. After displacing it from equilibrium and releasing it from rest, how long does it take to cut the whole pineapple? Neglect friction.



Pineapple Slicer

(A) 14.27 s (B) 28.52 s (C) 29.27 s (D) 40.33 s (E) 57.79 s

19. A thin drawbridge of uniform mass density, mass M, and length L is connected to a wheel of radius R, which is attached to pulleys. Both pulleys are frictionless and have a mass of $M/12$. The string wraps around the top pulley, under the bottom pulley, and over the top pulley again. What value of m minimizes the maximum force a motor has to exert on m (in any direction) to raise the drawbridge to an angle of 60° above the horizontal and lower it back down to horizontal?

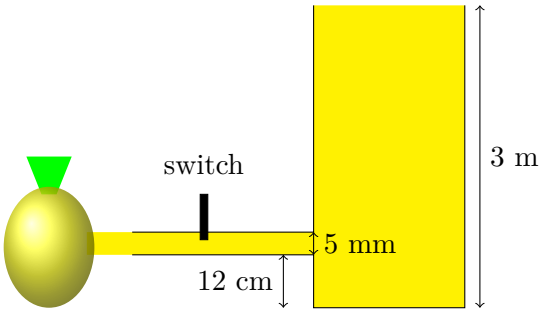


Drawbridge

(A) $\frac{M}{16}\left(\frac{L}{R}-\frac{1}{3}\right)$ (B) $\frac{M}{16}\left(\frac{3L}{2R}-\frac{1}{3}\right)$ (C) $\frac{M}{8}\left(\frac{L}{R}-\frac{1}{6}\right)$

(D) $\frac{M}{8}\left(\frac{3L}{2R}-\frac{1}{3}\right)$ (E) $\frac{M}{4}\left(\frac{L}{R}-\frac{1}{6}\right)$

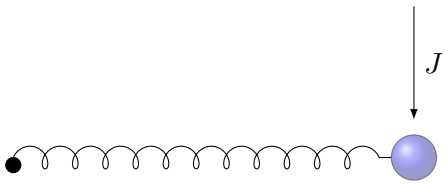
20. Elppaenip builds a pineapple juice gun with a 3 m tall reservoir and a 5 mm diameter nozzle at height 12 cm with a switch. A pineapple is directly in front of the nozzle. The juice comes to rest when it hits the pineapple. What is the force on the pineapple immediately after Elppaenip opens the switch? Assume the nozzle is much smaller than the reservoir. The density of pineapple juice is 1042 kg/m^3.



Juice Gun

(A) 1.18 N (B) 1.23 N (C) 2.26 N (D) 2.36 N (E) 2.46 N

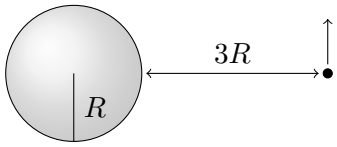
21. A massless spring with zero relaxed length and spring constant k is attached to a fixed pivot and a mass m. It oscillates one-dimensionally on a flat frictionless surface with amplitude A. When the mass is motionless, an impulse J is delivered perpendicular to the spring. Find the area enclosed by the particle's path.



Oscillating Impulse

(A) $\frac{\pi}{4}AJ\sqrt{\frac{1}{mk}}$ (B) $\frac{\pi}{2}AJ\sqrt{\frac{1}{mk}}$ (C) $\pi AJ\sqrt{\frac{1}{2mk}}$ (D) $\pi AJ\sqrt{\frac{1}{mk}}$ (E) $\pi AJ\sqrt{\frac{2}{mk}}$

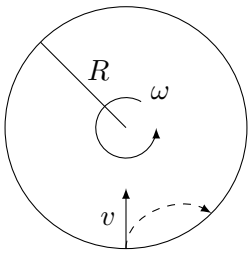
22. A satellite is launched from the surface of a spherical planet of mass M and radius R. What is the minimum launch speed v so that it reaches a distance $3R$ from the surface and does not subsequently collide with the planet? Figure not drawn to scale.



Orbital Transit

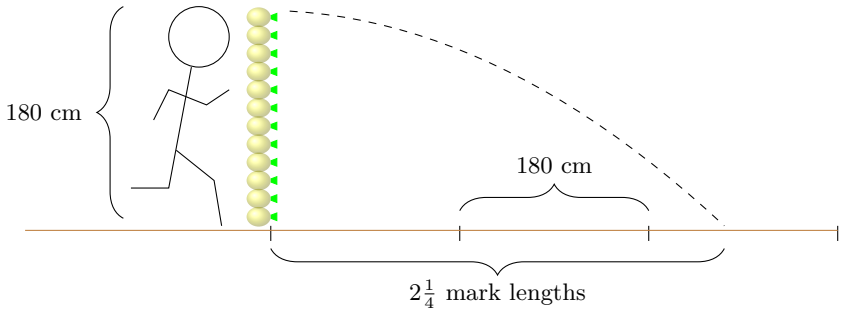
(A) $\sqrt{\frac{3GM}{4R}}$ (B) $\sqrt{\frac{4GM}{3R}}$ (C) $\sqrt{\frac{3GM}{2R}}$ (D) $\sqrt{\frac{8GM}{5R}}$ (E) $\sqrt{\frac{7GM}{4R}}$

23. Alice lives in a giant cylindrical space station of radius R rotating with angular velocity ω. She jumps straight up with velocity $v = \frac{\sqrt{3}}{3}R\omega$. With what speed, relative to the surface, does she land?



(A) $\frac{\sqrt{3}}{3}R\omega$ (B) $\frac{\sqrt{2}}{2}R\omega$ (C) $\frac{\sqrt{3}}{2}R\omega$ (D) $R\omega$ (E) $\frac{3+\sqrt{3}}{3}R\omega$

24. Elppaenip is stuck on a sandy desert island with a dozen identical pineapples and measures their mass. He has mass 60 kg, height 180 cm, and top sprinting speed 8.1 m/s. He measures marks in the sand using his height. He stacks the pineapples to his height, sprints towards them, and collides inelastically with them. The top pineapple travels $2\frac{1}{4}$ marks. All values except g have a 4% uncertainty. After the collision, Elppaenip does not touch the pineapples and they have equal velocity. Which of the following will reduce the measurement error of the mass of one pineapple the most? Neglect air resistance.

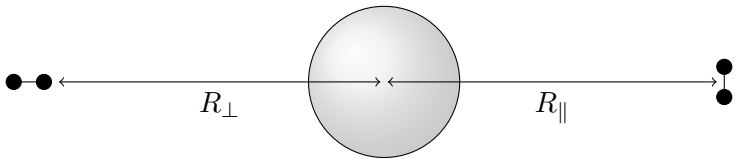


Pineapple Island

(A) Elppaenip knows his height exactly
(B) Elppaenip knows his mass exactly
(C) Elppaenip brings a ruler to measure all lengths exactly
(D) Elppaenip brings a radar gun to measure his running speed exactly
(E) Elppaenip performs the same experiment with two adjacent stacks of pineapples (of the same height)

25. Two masses are connected by a massless rigid rod of length ℓ, which breaks under tension or compression forces exceeding F. Let $R_\parallel$ and $R_\perp$ be the distance to a spherical planet when the rod breaks parallel and perpendicular to the surface, respectively. Find $\dfrac{R_\perp}{R_\parallel}$. Assume $R_\parallel \gg \ell$ and $R_\perp \gg \ell$.

The following may be useful: for $x \ll 1$, $(1+x)^k \approx 1 + kx$.



Fragile Rod

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{\sqrt[3]{2}}$ (C) 1 (D) $\sqrt[3]{2}$ (E) $\sqrt{2}$