PhODS MOCK PHYSICS COMPETITION 2024

2024 $F = ma$ Exam Solutions

#	Problem	Answer	Difficulty	Concept(s)
1	Book on Table	B	*	Forces
2	Block Collision	D	*	Energy, Collisions
3	Friction $v(x)$ Graph	C	*	Forces, Kinematics
4	Power of Car	C	*	Work, Energy, Power
5	Cow in Trailer	C	*	Forces
6	Midair Collision	E	*	Kinematics
7	Cow Carousel	B	*	Angular Momentum
8	Wind Power	E	*	Dimensional Analysis
9	Drown or Float	A	**	Fluid Statics
10	Elppaenip's Pineapple	E	**	Forces
11	Pineapple Truck	A	**	Forces, Momentum
12	PhODS-land Gravity	E	**	Gravitation
13	Inertia Comparison	C	**	Moment of Inertia's
14	Power-Energy Graph	B	**	Work, Power
15	Timmy Tipping	A	**	Statics
16	Bullet Hits Slab	D	**	Angular Momentum, Collisions, Energy
17	Perpendicular Projectile	D	**	Kinematics
18	Pineapple Slicer	B	**	Oscillations
19	Drawbridge	B	***	Forces, Torques
20	Juice Gun	A	***	Fluid Dynamics, Momentum
21	Zero Length Ellipse	D	***	Energy, Momentum
22	Orbital Transit	D	***	Orbital Mechanics
23	Space Station	A	***	Rotational Kinematics
24	Pineapple Island	E	***	Error Propagation, Collisions, Kinematics
25	Rod Tidal Forces	D	***	Gravity, Statics

The stars indicate a problem's difficulty. One-star problems are meant to be solvable with a strong understanding of mechanics at the AP Physics 1 level. Two-star problems are trickier, combining multiple concepts and/or requiring deeper understanding. Three-star problems are the most complex or tricky problems, requiring significant calculation or insight.

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1 Book on Table

The two forces acting on the block are the normal force N and the friction force $f = \mu N$. Since they are orthogonal, the magnitude of their sum can be computed with the Pythagorean theorem which gives

$$\left|\vec{N} + \vec{f}\right| = N\sqrt{1+\mu^2}$$

From force balancing we have $N = mg$ so the magnitude is

$$mg\sqrt{1+\mu^2} = \boxed{45 \text{ N}}$$

Note that the force exceeds the maximum value of static friction so the block is sliding.

2 Block Collision

Let m_1 and m_2 be the masses of the 2 and 5 kg blocks, respectively, and l be the length of the ramp. By energy conservation, the velocity v of m_1 at the bottom of the ramp can be calculated.

$$m_1 g l \sin(30^\circ) = \frac{1}{2} m_1 v^2$$

$$v^2 = gl$$

After finding v, we can use momentum conservation to find the final velocity u of the blocks after the collision.

$$m_1 v = (m_1 + m_2)\, u$$

$$u = \frac{m_1\sqrt{gl}}{m_1 + m_2}$$

Finally, we subtract the final kinetic energy from the initial potential energy.

$$m_1 g l \sin(30^\circ) - \frac{1}{2}(m_1 + m_2) u^2 = \boxed{42.9 \text{ J}}$$

3 Friction $v(x)$ Graph

First, note that (A) and (D) are impossible because the friction force opposes the block as the block moves forward, thus decreasing the velocity. We know that the acceleration of the block is constant, as the force of kinetic friction is constant and no other forces act on the block. We can also notice that the velocity of the block decreases with its distance, thus, the block must spend more time at each small interval Δx. As the change in velocity for each Δx is proportional to the time spent to cover the distance, we can expect the deceleration as a function of x to increase. The only graph with this feature is $\boxed{\text{(C)}}$.

4 Power of Car

The energy output is the kinetic energy of the car when it moves at 90 km/h plus the work done to overcome the retarding force. The work done by the retarding force is the distance traveled times the force. The distance traveled is $\frac{1}{2}vt$ where t is the time and the work is $\frac{1}{2}Fvt$ where F is the retarding force. The total energy output is

$$E = \frac{1}{2}(M + 4m)v^2 + 4 \cdot \left(\frac{1}{2}I\omega^2\right) + \frac{1}{2}Fvt$$

where v is the velocity of the car, M is the mass of the car, m is the mass of one wheel, I is the moment of inertia of one wheel, and ω is the angular velocity of the wheels. The moment of inertia of a uniform disc is $\frac{1}{2}mr^2$. By the non-slipping condition, the angular velocity is $\omega = \frac{v}{r}$. Plugging those values in,

$$E = \frac{1}{2}(M + 4m)v^2 + mv^2 + \frac{1}{2}Fvt$$

The power is

$$P = \frac{E}{t} = \boxed{46000 \text{ Watts}}$$

5 Cow in Trailer

Since the cow and the trailer move together, Newton's second law gives

$$F = (M + m_{\text{cow}})\,a$$

We can calculate the mass of the cow by calculating the volume of a sphere and multiplying by the density

$$m_{\text{cow}} = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3\rho = \frac{\pi h^3 \rho}{6}$$

Thus, we can see that our final answer is $\boxed{\left(M + \frac{\pi\rho h^3}{6}\right)a}$.

6 Midair Collision

Since the particles are launched with the exact same initial parameters, their vertical positions will always be identical and their horizontal velocity will be the same in opposite directions. This means that when they collide, the impulse is completely horizontal. Thus, they will continue at their identical vertical velocities and land at the same time, so statement I is true. Since the collision does not change their vertical velocity, they will also land at the same time that they would have landed without the collision, so statement III is true. Since horizontal momentum is conserved, the masses end up with the same magnitude of horizontal velocity and they will land the same distance from M, so statement III is also true. The final answer is $\boxed{\text{(E)}}$.

7 Cow Carousel

The angular momentum of the carousel with the cows is its moment of inertia multiplied by its angular velocity. The moment of inertia of each cow is $mr^2 = i^2\left(i^3\right)^2 = i^8$ so the total moment of inertia is proportional to N^9. Since the outermost cow is N^3 meters from the center and traveling at N^4 meters per second, the angular velocity of the carousel is $\frac{v}{r} = N$. Therefore, the final result for the angular momentum is $\boxed{N^{10}}$.

8 Wind Power

The units for power are $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$. Note that ρ has units of $\frac{\text{kg}}{\text{m}^3}$, L has units of m, and v has units of $\frac{\text{m}}{\text{s}}$. By dimensional analysis, we can see that the only answer choice with the correct units is $\boxed{k\rho v^3 L^2}$.

9 Drown or Float

Let V_0 be the total volume of the cylindrical object and V be the volume displaced when it floats. Equating the weight of the displaced water and the object, we get that when it floats,

$$\rho_c V_0 g = \rho_w V g$$

Solving for V_0,

$$V_0 = \frac{\rho_w V}{\rho_c}$$

$$\pi R^2 (H - h) = \frac{\rho_w \pi R^2 y}{\rho_c}$$

Solving for H,

$$H = \boxed{h + \frac{\rho_w}{\rho_c} y}$$

10 Elppaenip's Pineapple

When Elppaenip is bending down, his center of mass (CM) accelerates downward, thus decreasing the normal force from the scale and the scale reading. His CM now has a downward velocity. In order to come to rest and pick up the pineapple, his CM must experience an upward acceleration to cancel out the downward velocity, so we expect a positive spike in the scale reading with a magnitude larger than Elppaenip's weight in equilibrium. We would expect a similar positive spike when he stands back up as his CM accelerates upward, and a negative spike when the normal force decreases to allow his CM to come to rest. The only graph with these features is $\boxed{\text{(E)}}$.

11 Pineapple Truck

The sum over time of the forces on the pineapples is equal to the momentum of the pineapples. Since the pineapples start at rest and end at rest, the average net force on them must be zero, and the force from the bridge must cancel out the gravitational force. Therefore, statement I is true. Before any pineapples hit the ground, the force on the bridge is less than $(M + m)\,g$. Therefore, for the average to be $(M + m)\,g$, the maximum force must be greater than $(M + m)\,g$, and statement II is false. The horizontal impulse required to stop the pineapples is equal to the momentum of the pineapples, mv, but this impulse is applied by both the bridge and air resistance. Therefore, the impulse applied to the bridge is less than mv and statement III is false. Finally, the answer is $\boxed{\text{(A)}}$

12 PhODS-land Gravity

The gravity of PhODS-land can be modeled as that of a solid sphere of radius R of the same density and two spheres of radius $R/2$ of negative density located at $y = \pm R/2$. The mass of the large sphere is $\frac{4M}{3}$ and the mass of each small sphere is $-\frac{M}{6}$. The total gravitational field is therefore

$$\begin{aligned} g &= \frac{4GM}{3\left(\sqrt{2}R\right)^2} - 2\frac{GM}{6\left(\frac{3}{2}R\right)^2}\frac{\sqrt{2}}{\frac{3}{2}} \\ &= \frac{2GM}{3R^2} - \frac{8GM}{27R^2}\frac{\sqrt{2}}{3} \\ &= \boxed{\frac{2GM}{3R^2}\left(1 - \frac{4\sqrt{2}}{27}\right)} \end{aligned}$$

13 Inertia Comparison

By the perpendicular-axis theorem, I_a is equal to the sum of I_b and the moment of inertia about an axis in the plane of the shape perpendicular to I_b. Therefore, I_a must be greater than I_b. By the parallel-axis theorem, $I_c = I_a + Md^2$ where M is the mass of the shape and d is the distance between the axes. This means that I_c must be greater than I_a, leaving the correct answer to be $\boxed{\text{(C)}}$.

14 Power-Energy Graph

The momentum of the particle is equal to the force multiplied by the time, which is the area under the graph. At $t = 4$, the area under the graph is 48 kg m/s. Dividing by the mass, we get a velocity of 12 m/s. Multiplying by the force, the power at $t = 4$ is 192 W. At $t = 8$, the area under the graph is 98 kg m/s, which means the velocity is 24.5 m/s and the power is 294 W. Therefore, the power at $t = 4$ is less than the power at $t = 8$, leaving only choices A and B. Since the force remains positive under $t = 10$, the kinetic energy continues increasing and thus, the correct answer is $\boxed{\text{(B)}}$.

15 Timmy Tipping

First, we must find the magnitude of F. When the chair barely tips back, all the normal force is on the back leg so we balance torques about this point. Setting the tip of the back leg as the origin, we can find the center of mass to be at $\left(\frac{3}{8}a, \frac{7}{8}a\right)$. The mass of the chair is $4m$, so the torque is $\frac{3}{2}mga$. Timmy applies a torque of equal magnitude at a distance $\frac{3}{2}a$, and dividing, we get $F = mg$.

Next, for the modified chair, we set our origin to the new back leg. The CM of the new chair will be $\frac{1}{3}(0, a) + \frac{2}{3}\left(\frac{3}{8}a + x, \frac{7}{8}a\right)$. Thus, gravity applies a torque of $6mg\left(\frac{1}{4}a + \frac{2}{3}x\right)$. The maximum torque from a force F is if we apply it at the top of the chair, at height $2a$. The torque will then be $2mga$. Balancing torques, we have $6mg\left(\frac{1}{4}a + \frac{2}{3}x\right) = 2mga$. Solving for x, $x = \boxed{\frac{1}{8}a}$

16 Bullet Hits Slab

During the collision, the angular momentum about the end of the slab on the ledge is conserved. After the collision, the kinetic energy is conserved. Let I be the moment of inertia of the slab and bullet about the pivot.

$$I\omega = mv\ell$$
$$\frac{1}{2}I\omega^2 = \frac{Mg\ell}{2} + mg\ell$$
$$\frac{(mv\ell)^2}{2\left(\frac{M\ell^2}{3} + m\ell^2\right)} = \frac{Mg\ell}{2} + mg\ell$$
$$\implies v = \boxed{\sqrt{\frac{g\ell(M+2m)(M+3m)}{3m^2}}}$$

Note that linear momentum is not conserved, as the floor exerts a normal force on the slab during the collision. Since the collision is fast, the torque from gravity can be neglected.

17 Perpendicular Projectile

Let us use a coordinate system tilted along the inclined plane. In this coordinate system, gravity acts at an angle of 45° to the horizontal. Let g' be the equal horizontal and vertical components of the gravitational force in this frame, v_x be the initial horizontal velocity, v_y be the initial vertical velocity, and t be the flight time. The projectile must have zero horizontal velocity to land perpendicular to the plane. The vertical velocity component behaves in the same way as a normal projectile motion, which means the projectile will land with vertical velocity $-v_y$.

$$v_x - g't = 0$$
$$v_y - g't = -v_y$$
$$\implies v_x = 2v_y$$

The angle to the inclined plane is therefore $\arctan\left(\frac{v_y}{v_x}\right) = \arctan\left(\frac{1}{2}\right) \approx 27°$. Adding that to the 45° incline, the final answer is $\boxed{72°}$.

As an exercise, one could also take this scenario to show that for a plane of angle θ, the angle from a horizontal a projectile needs to be thrown to satisfy the perpendicular condition is given by

$$\phi = \theta + \arctan\left(\frac{1}{2\tan\theta}\right)$$

For this case, plugging in $\theta = 45°$ yields the same result as above.

18 Pineapple Slicer

The two springs have a combined spring constant of $2k$ (since each spring exerts a force kx on the pineapple). The time between each slicing is half of the oscillation period, $\pi\sqrt{\frac{m}{2k}}$. Since the mass changes by a factor of 0.9 for each slice, the total time is

$$T = \sum_{n=0}^{\infty} \pi\sqrt{\frac{0.9^n m}{2k}} - \frac{\pi}{2}\sqrt{\frac{m}{2k}}$$

The second term accounts for the pineapple only moving for half a cycle after it is first released. Solving and plugging in numbers,

$$T = \frac{\pi}{2}\sqrt{\frac{m}{2k}}\left(\frac{2}{1-\sqrt{0.9}} - 1\right)$$

$$T = \boxed{28.2\text{ s}}$$

19 Drawbridge

Since the system moves slowly, we can approximate it as static. The tension in the string must balance out the weight of the mass and the motor force, $mg + F$. The string wraps four times around the top pulley, so the force on the top pulley is $4(mg + F)$ and the torque on the wheel is $R\left(4mg + 4F + \frac{Mg}{12}\right)$, accounting for the weight of the pulley. The torque on the wheel necessary to lift the bridge is $\frac{MgL}{2}$ when the bridge is flat and $\frac{MgL}{4}$ when the bridge is lifted to 60°. The best scenario is when the motor is exerting the same force in opposite directions in the two scenarios.

$$R\left(4mg + 4F + \frac{Mg}{12}\right) = \frac{MgL}{2}$$

$$R\left(4mg - 4F + \frac{Mg}{12}\right) = \frac{MgL}{4}$$

Subtracting the two equations from each other,

$$8F = \frac{MgL}{4R}$$

Plugging back in,

$$4mg + \frac{MgL}{8R} + \frac{Mg}{12} = \frac{MgL}{2R}$$

$$m = \boxed{\frac{M}{16}\left(\frac{3L}{2R} - \frac{1}{3}\right)}$$

20 Juice Gun

Using Bernoulli's equation on the juice,

$$\rho g (H - h) = \frac{1}{2}\rho v^2$$

where H is the height of the reservoir, h is the height of the nozzle, and v is the speed at which the juice exits the nozzle. Solving for v,

$$v = \sqrt{2g(H - h)}$$

During a small time dt, a mass dm of pineapple juice transfers all of its mass to the pineapple. Equating the impulse and momentum,

$$Fdt = vdm = v(\rho A v)$$

$$F = \frac{\pi d^2 \rho g (H - h)}{2} = \boxed{1.18\text{ N}}$$

21 Zero Length Ellipse

For convenience, let $v \equiv \frac{J}{m}$ be the velocity from the impulse. Energy is conserved. Angular momentum about the pivot is conserved because the spring force is always along the line of action of the mass. At some point the mass will be traveling perpendicular to the spring at distance r; by angular momentum conservation, its speed at that point will be $v\frac{A}{r}$. By conservation of energy,

$$\frac{1}{2}mv^2 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2\frac{A^2}{r^2} + \frac{1}{2}kr^2$$

Solving for r,

$$r = v\sqrt{\frac{m}{k}}$$

The area of an ellipse is $ab\pi$ where a and b are the semimajor and semiminor axes, which are A and $v\sqrt{\frac{m}{k}}$, giving us our answer, $\boxed{AJ\sqrt{\frac{1}{mk}}\pi}$.

An alternative solution can be found by recognizing that the spring force, $\vec{F} = -k\vec{r}$, can be separated into $F_x = -kx$ and $F_y = -ky$. If we assume that initially the spring was oscillating on the x-axis and suddenly acquires velocity v along the y-axis, the amplitude of oscillations in the y-direction will be v/ω, where $\omega = \sqrt{\frac{k}{m}}$. This gives the same result.

Yet another solution can be found by observing Kepler's second law, which states that the area swept out in constant times is equal, is simply a statement of angular momentum conservation and therefore applies here. The amount of area swept per unit time is $\frac{Av}{2}$, and the period of the orbit is $2\pi\sqrt{\frac{m}{k}}$. Multiplying yields the same result.

22 Orbital Transit

Apply conservation of angular momentum and energy when the satellite is first launched and when it reaches its farthest distance $4R$ from the center of the planet. Let v be the launch velocity and u be the velocity at apoapsis.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mu^2 - \frac{GMm}{4R}$$
$$mvR = 4muR$$

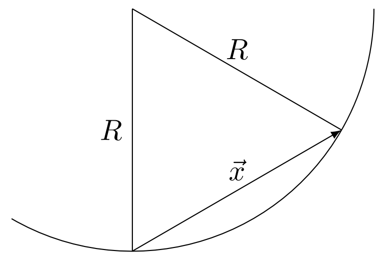
Solving the second equation for u and plugging into the first,

$$\frac{1}{2}v^2 - \frac{1}{2}\left(\frac{v}{4}\right)^2 = \frac{GM}{R} - \frac{GM}{4R}$$

$$v = \boxed{\sqrt{\frac{8GM}{5R}}}$$

23 Space Station

It is easiest to solve this problem in an inertial reference frame. Observe that in this frame, she jumps from and lands at two different points both a distance R from the center of the space station and she travels in a straight line with constant speed (since no forces act on her during the jump). The two radii and her displacement $\vec{x}$ form an isosceles triangle. Therefore, she jumps and lands at the same angle relative to the space station and same speed, which means that her velocity relative to the space station must also be the same at $\boxed{\frac{\sqrt{3}}{3}R\omega}$.



24 Pineapple Island

Let M be Elppaenip's mass, m be the mass of a pineapple, h be Elppaenip's height, v be his running speed, N be the number of marks that the top pineapple travels and u be the speed of the top pineapple immediately after the collision. For the projectile motion of the top pineapple,

$$h = \frac{1}{2}gt^2$$
$$Nh = ut$$

Solving the first equation for t, substituting into the second equation, and solving for u,

$$u = Nh\sqrt{\frac{g}{2h}}$$

By conservation of momentum,

$$Mv = (M + 12m)\,u$$
$$m = \frac{M\left(\frac{v}{N}\sqrt{\frac{2}{gh}} - 1\right)}{12} = 1 \text{ kg}$$

Let $x \equiv \frac{v}{N}\sqrt{\frac{2}{gh}}$. Then,

$$\frac{\delta x}{x} = \sqrt{0.04^2 + 0.04^2 + (0.5 \times 0.04)^2} = .06$$

Knowing h will eliminate a 2% error, knowing v will eliminate a 4% error, and having the ruler (knowing both h and N) will eliminate a 4% and a 2% error. Therefore, we see qualitatively that out of these three, the ruler reduced the error the most and we can eliminate (A) and (D) without calculations.

Now, observe that when subtracting by 1, the absolute uncertainty does not change but the percent uncertainty greatly increases. Therefore, the error in $x - 1$ far exceeds the error in M, and eliminating the error in M has negligible effects which means (B) can be eliminated. Now we calculate and compare the percent errors in the $x-1$ term in the scenarios in (C) and (E). For the setup in (C),

$$\frac{\delta(x-1)}{x-1} = \frac{.04x}{x-1} = .04\frac{1.2}{0.2} = .24$$

Let u' be the speed of the top pineapple immediately after the collision in setup (E). Since the same amount of momentum enters (C) and (E),

$$\begin{aligned}(M+12m)\,u &= (M+24m)\,u' \\ u' &= \frac{M+12m}{M+24m}u \\ &= \frac{6}{7}u\end{aligned}$$

Since N is directly proportional to u and x is inversely proportional to N, $x' = \frac{7}{6}x = 1.4$.

$$\frac{\delta(x'-1)}{x'-1} = \frac{.06x'}{x'-1} = .06\frac{1.4}{0.4} = .21$$

Therefore, our final answer is that $\boxed{\text{(E)}}$ reduces the error the most.

25 Rod Tidal Forces

Let the center of the planet be at the origin. Consider the perpendicular case first. The masses are at $(R_\perp, 0)$ and $(R_\perp + \ell, 0)$ and experience forces $\frac{GMm}{R_\perp^2}$ and $\frac{GMm}{(R_\perp+\ell)^2}$, respectively. The tension in the rod is the difference between these two forces.

$$\begin{aligned}T &= \frac{GMm}{R_\perp^2} - \frac{GMm}{(R_\perp+\ell)^2} \\ &= \frac{GMm}{R_\perp^2} - \frac{GMm}{R_\perp^2\left(1+\frac{\ell}{R_\perp}\right)^2}\end{aligned}$$

Using the binomial approximation,

$$\begin{aligned}T &= \frac{GMm}{R_\perp^2} - \frac{GMm}{R_\perp^2}\left(1 - \frac{2\ell}{R_\perp}\right) \\ &= \frac{2\ell GMm}{R_\perp^3}\end{aligned}$$

Now consider the parallel case. The masses are at $(R_\parallel, 0)$ and $(R_\parallel, \ell)$. The radial components of the forces are identical but the mass at $(R_\parallel, \ell)$ has a tangential component of force. The compression in the rod must cancel out that tangential force so

$$T = \frac{GMm}{R_\parallel^2}\frac{\ell}{R_\parallel} = \frac{\ell GMm}{R_\parallel^3}$$

Setting the two tensions equal to each other,

$$\frac{2\ell GMm}{R_\perp^3} = \frac{\ell GMm}{R_\parallel^3}$$
$$\frac{R_\perp}{R_\parallel} = \boxed{\sqrt[3]{2}}$$