Circuits Mock Exam

10 MCQ 2 FRQ - 60 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 9.8$ N/kg throughout this contest.
- Test under standard conditions, which means that you must complete the test in 60 minutes in one sitting.
- This test contains 10 Multiple Choice Questions and 2 Free Response Questions.
- Correct answers will be awarded the points shown; leaving an answer blank will be awarded zero points. The amount of partial credit is up to your discretion as the grader. Our recommendation is to be more strict than necessary.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a graphing calculator. Calculators may not be shared. Cell phones may not be used during the exam. You may not use tables, books, or formula collections.
- The number in **red** next to each question represents the amount of points the question is worth. There are a total of 40 points on this test.
- If you have any questions or clarifications, please contact us at tjhsstphysicsteam@gmail.com.

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Good luck and have fun! Turn the page to start the mock.

Fundamental Constants

$$e = 1.602 \times 10^{-19}\,\mathrm{C}$$
$$\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{N\,m^{-1}}$$
$$\pi = 3.141$$

10 MCQs

[2] **Problem 1.** What is the equivalent resistance of the following setup?

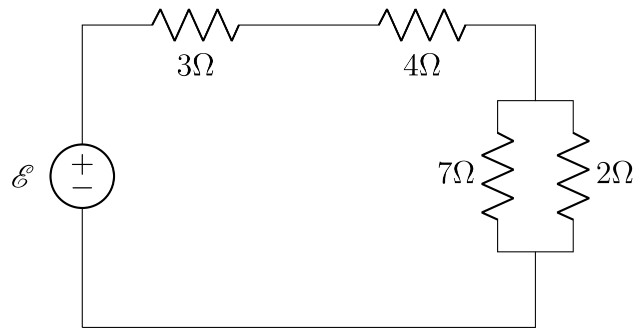


Figure 1: Resistors in a Circuit

A) 6.12Ω

B) 8.56Ω

C) 9.89Ω

D) 10.45Ω

E) 16.00Ω

Solution. Combine the resistance of the resistors in parallel first:

$$R_{27} = \left(\frac{1}{7} + \frac{1}{2}\right)^{-1} = 1.56\Omega$$

Then, add the resistors in series:

$$R_{\text{eff}} = 3\Omega + 4\Omega + 1.56\Omega = 8.56\Omega$$

[2] **Problem 2.** Suppose that circuits A and B are both DC circuits, each with an ideal battery of voltage V. Circuit A contains two resistors, each of resistance R, added in parallel, while circuit B contains two resistors of resistance R added in series. If P is the power dissipated as heat by the entire circuit, what is the ratio of P_A/P_B? Suppose that the wires have negligible resistance.

A) 1/16

B) 1/4

C) 1/2

D) 1

E) 2

F) 4

G) 16

Solution. Circuit A has an effective resistance of R/2, while circuit B has an effective resistance of 2R, and both circuits are equivalent to a battery connected to a single resistor with the correct effective resistance. For both of these circuits, the batteries have a voltage V, so the potential difference across the resistor is -V. The power dissipated by the resistor is $\frac{(\Delta V)^2}{R_{eq}} = \frac{V^2}{R_{eq}}$. Therefore,

$$\frac{P_A}{P_B} = \frac{\frac{V^2}{R_{eq,A}}}{\frac{V^2}{R_{eq,B}}} = \frac{R_{eq,B}}{R_{eq,A}} = \frac{2R}{R/2} = 4$$

Answer: $\boxed{F}$ Note: Be careful when trying to use Ohm's law $I = \Delta V/R$ or its equivalent forms because while ΔV across the entire equivalent resistor is held constant (due to the battery operating at constant emf), the current in the circuit need not be constant. In reality, the current that leaves the battery in Circuit A is four times that of circuit B! Attempting to use $P = i^2R$ and assuming that i is the same in the two circuits leads to the *incorrect answer* of 1/4, or B. When comparing two circuits with different resistances, pay extra attention to which values are constant and which ones are not.

[2] **Problem 3.** When an RC circuit with resistance R and capacitance C with resistor and capacitor *in series* is attached to a battery of voltage V, it takes time T to charge the capacitor to 50% of it's maximum charge. In an RC circuit with a battery of voltage $2V$ and with the resistor and capacitor *in parallel*, how long will it take for the capacitor to gain the same charge as the capacitor in the first circuit?

A) The capacitor will appear to charge instantly

B) $\frac{1}{2}T$

C) T

D) $2T$

E) The charge on the capacitor will never reach this amount

Solution. The branch of the circuit containing the capacitor has no resistor, so the resistance is close to 0 (in a real setting it would not be exactly 0 since there is still a small resistance in the wires). Therefore, all of the current will pass through this branch rather than the branch with the resistor, and the current will be much higher than the current in the first circuit. Therefore, the capacitor will charge to it's maximum charge ($Q = 2CV$), nearly instantly. The problem asks for the time it takes for the capacitor to gain half of the maximum charge from the other circuit, which is $\frac{1}{2}CV$, which is also close to 0. Therefore, the answer is $\boxed{A}$.

[2] **Problem 4.** A capacitor with capacitance C is charged using a battery of voltage V_0. The capacitor is then disconnected from the battery, and a dielectric is inserted into the capacitor, which effectively doubles its capacitance. How much work was done on the system when inserting the dielectric?

A) $-\frac{1}{2}CV_0^2$

B) $-\frac{1}{4}CV_0^2$

C) 0

D) $\frac{1}{4}CV_0^2$

E) $\frac{1}{2}CV_0^2$

Solution. Since $C = \frac{Q}{v_0}$, where Q is the charge on the plates of the capacitor, $Q = CV_0$. Since the potential difference across the charged capacitor is V_0, the initial potential energy stored in the capacitor is $U_i = \frac{1}{2}CV_0^2 = \frac{Q^2}{2C}$. When the capacitor is not connected to the battery and the dielectric is inserted, the charge on the capacitor plates is constant, so the final potential energy is $U_f = \frac{Q^2}{2(2C)} = \frac{Q^2}{4C}$. Using $Q = CV_0$, we find that $U_f = \frac{1}{4}CV_0^2$. The work done on the system equals the change in potential energy of the capacitor, which is $\Delta U = U_f - U_i = -\frac{1}{4}CV_0^2$, or $\boxed{B}$.

[2] **Problem 5.** A spherical conducting shell has some "self-capacitance", which is the ratio between the charge it carries and the electric potential at its surface. In a shell with radius $R = 3$cm. What is the work done in charging the capacitor to a charge $q = 1.5 * 10^{-7}$C? Hint: First find the self-capacitance of a general sphere with radius r by finding the ratio $Q/\Delta V = Q/V$ (since potential is zero at infinite distance away) and use your capacitance and the given radius to evaluate the total potential energy.

A) $1.12 * 10^{-5}$ J

B) $3.37 * 10^{-3}$ J

C) $7.59 * 10^{-2}$ J

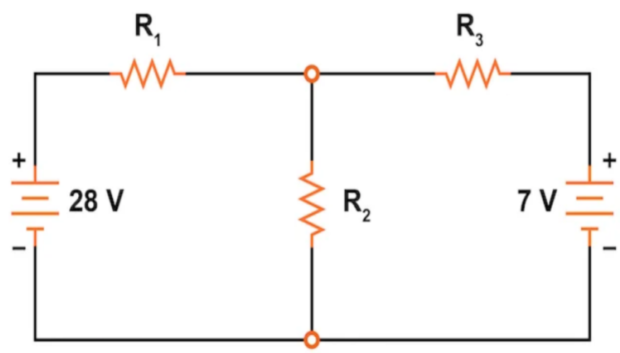
D) $9.65 * 10^{2}$ J

E) $5.33 * 10^{3}$ J

Solution. By the shell theorem, the conducting sphere is equivalent to a point charge with the same charge. Following Coulomb's Law, the electric potential at the surface of the sphere is written as $V = \frac{kQ}{r}$. The self capacitance of a conducting sphere is thus $C = \frac{Q}{V} = \frac{r}{k}$. The potential energy of a capacitor is $U = \frac{Q^2}{2C} = \frac{Q^2}{2C} = \frac{(1.5*10^{-7})^2}{2*\frac{0.03}{8.99*10^9}} = 3.37 * 10^{-3}$ J, or $\boxed{B}$

Note: if you are skeptical that the potential energy formula still applies to this system because there isn't a second shell (this is still technically a capacitor but with the second shell very far away), you can also use dU = V dq, plug in the formula for V in terms of q, and integrate as q goes from 0 to the final Q (this is effectively the same trick used when finding the potential energy stored in a parallel plate capacitor), which ends up leading to the same result.

[2] **Problem 6.** The DC circuit shown below has resistances $R_1 = 4\Omega$, $R_2 = 4\Omega$, and $R_3 = 5\Omega$. What is the absolute value of the potential difference across resistor R_2?



A) 5 V

B) 8 V

C) 12 V

D) 18 V

E) 24 V

Solution. Let I_1, I_2, and I_3 be the currents passing through resistors R_1, R_2, and R_3 respectively, where I_1 and I_2 are positive if the current moves to the right, and I_3 is positive if the current moves down (If any of these assumptions are wrong then we will just end up with a negative current). By Kirchhoff's junction rule, $I_1 = I_2 + I_3$ (Note: If you guessed different directions for I_1, I_2, or I_3 then your junction rule equation might be different)

Applying Kirchhoff's voltage law clockwise around the left and right loops gives the equations (remember that an emf in reverse reduces the potential by the emf's voltage):

$$28 - R_1I_1 - R_2I_2 = 28 - 4I_1 - 4I_2 = 0 \to I_1 + I_2 = 7$$

$$-R_3I_3 - 7 + R_2I_2 = -5I_3 - 7 + 4I_2 = 0 \to 4I_2 - 5I_3 = 7$$

Plugging in the Kirchhoff's junction rule equation into I_1 from the first equation,

$$2I_2 + I_3 = 7$$

Solving the system of equations:

$$(I_1, I_2, I_3) = (4\text{A}, 3\text{A}, 1\text{A})$$

Therefore, the potential drop across the R_2 resistor is $|\Delta V| = R_2I_2 = 12$ V Answer: $\boxed{C}$

[2] **Problem 7.** A cylindrical metal wire with a diameter of 3 mm and length 0.2 m conducts a current of 12 A. The wire is made of a uniform material and the electric field inside the wire is $2.75 * 10^{-2}$ N/C. Calculate the resistivity of the material.

A) $1.35 * 10^{-9}\ \Omega \cdot m$

B) $4.05 * 10^{-9}\ \Omega \cdot m$

C) $5.40 * 10^{-9}\ \Omega \cdot m$

D) $1.62 * 10^{-8}\ \Omega \cdot m$

E) $4.82 * 10^{-7}\ \Omega \cdot m$

F) $3.44 * 10^{-6}\ \Omega \cdot m$

Solution. When dealing with the microscopic nature of circuits, it can be helpful to use the alternative form of Ohm's law, which states that $E = \rho j$, where E is the magnitude of the electric field and j is the current density (defined as $j = I/A$ where A is the cross sectional area of the conductor). Thus,

$$\rho = \frac{E}{j} = \frac{E}{I/A} = \frac{\pi r^2 E}{I}$$

Plugging in our values, including $E = 2.75 * 10^{-2}$ N/C as given and r = 0.0015 m for the radius, we find $\rho = 1.62 * 10^{-8}$ $\boxed{D}$

Note: it was also possible to solve this problem by integrating E (which turns out to be constant anyways) along the length of the wire to find that $\Delta V = EL$, applying the macroscopic form of Ohm's Law $R = \frac{|\Delta V|}{I}$, and then using the resistance formula $R = \frac{\rho L}{A}$, which yields the same result for resistivity. However, this approach takes somewhat more work compared to applying $E = \rho j$, since we are not interested in finding the potential difference or the resistance. Also note that in the first solution, the length of the wire was not required, and in the alternate solution, this quantity would simply cancel out.

[2] **Problem 8.** Two conducting wires, both with the same density of electrons $8.49 * 10^{28}$ m^{-3} and a resistance of $4.5 * 10^{-2}\ \Omega$ maintain a potential difference of $9V$. However, one of the wires has a circular cross-section with diameter 5 mm, while the other has a square cross-section with a side length of 5 mm. What is the absolute value of the difference in electron drift velocities between the two wires, assuming that electrons are the only charge carriers in both wires? See the reference sheet for the charge of an electron.

Hint: The *charge density* in a conductor is equal to $q * n$, where q is the charge of the charge carriers and n is the density of the charge carriers.

A) $3.84 * 10^{-5}$ m/s

B) $1.61 * 10^{-4}$ m/s

C) $5.89 * 10^{-4}$ m/s

D) $7.50 * 10^{-4}$ m/s

E) $2.90 * 10^{-3}$ m/s

Solution. The formula for the current in a wire is $I = qnAv_d$, where A is the cross sectional area, v_d is the drift velocity, and qn is the charge density of the wire (you can rederive this by drawing a short segment of the wire and reasoning that in a short time Δt, the volume passing through a fixed cross-section is $Av_d\Delta t$, so multiplying this by charge density gives the total charge passing through this face in the same time: $\Delta Q = qnAv_d\Delta t$, so it follows that $I = \frac{\Delta Q}{\Delta t} = qnAv_d$

Rearranging: $v_d = \frac{I}{qnA} = \frac{|\Delta V|/R}{qnA} = \frac{|\Delta V|}{qnAR}$. Calculating the cross-sectional areas, the wire with a circular cross-section has an area of $A = \pi r^2 = \pi d^2/4 = \frac{\pi(5*10^{-3})^2}{4} = 1.963 * 10^{-5}$ m^{-2}. The wire with the square cross-section has an area of $A = s^2 = (5 * 10^{-3})^2 = 2.5 * 10^{-5}$ m^{-2}. Plugging these results into the drift velocity formulas, v_d is $7.5 * 10^{-4}$ m/s for the circular wire and $5.89 * 10^{-4}$ m/s for the square wire. The difference in velocities is then $1.61 * 10^{-4}$ m/s, or $\boxed{B}$

Note: Although it was not asked for, we can also check that the sign of the difference in velocities is what we would expect. The circular wire had a higher drift velocity compared to the square wire, which makes sense because current must be the same for the two wires, and based on the dimensions given, the circular wire has less cross-sectional area, so electrons must flow faster in order to keep the rate of charge flow the same (See section 16-2 of HRK volume 1 for an interesting application of a similar idea in fluids).

[2] **Problem 9.** Eric has stolen Aarush's phone! To Aarush's dismay, Eric used up all the battery playing Brawl Stars. Aarush, who now yearns to play Brawl Stars himself, begins to charge his phone with a very short 3.1 cm USB-C cable that carries a current of 19.6 A with a thick internal wire of diameter 18.7 mm. Assuming the phone begins to charge when electrons reach the phone after physically drifting from the adapter to the phone, how long would it take for the phone to start charging after Aarush plugs it in?

Use $8.49 * 10^{28}$ electrons/m^3 as the density of free electrons in copper.

A) 13.6 minutes

B) 31.2 minutes

C) 37.8 minutes

D) 72.1 minutes

E) 98.5 minutes

Solution. We start by calculating the drift velocity of the electrons in the wire, using the formula $v_d = \frac{I}{neA}$, where I is the wire current, n is the free electron density, e is the charge of an electron, and A is the cross sectional area. We initially solve for the cross sectional area as $A = \pi(\frac{0.0187\text{m}}{2})^2 = 2.746 * 10^{-4} m^2$. Plugging in the values we are given, we find

$$v_d = \frac{19.6A}{(8.4910^{28}\text{electrons}/m^3)(1.602 * 1019C)(2.746 * 10^{-4}m^2)} = 5.247 * 10^-6\text{m/s}$$

We then use $v = \frac{d}{t}$ to solve for t. Rearranging, we solve for t, we have $t = \frac{0.031m}{5.247*10^{-}6m/s} =$ 5908 seconds, or 98.5 minutes - $\boxed{E}$.

Note: This is obviously much longer than the practical charging time, even with exaggerated conditions of the wire and its characteristics. In reality, charging happens almost instantly because the voltage propagates through the wire at nearly the speed of light, and the drift velocity of the wire's electrons is much slower than the speed at which the electrical signal travels.

[2] **Problem 10.** When electric field is constant, which of the following is current density inversely proportional to?

A) Electric Potential

B) Current

C) Resistance

D) Resistivity

E) Voltage

F) None of the above

Solution. Ohm's law states that $V = IR$. We know $R = \frac{\rho L}{A}$, $E = \frac{V}{L}$, and $J = \frac{I}{A}$. Making rearrangements and substitutions, we have $V = \frac{I\rho L}{A}$, $E = \frac{V}{L} = \frac{I\rho}{A} = J\rho$. We end up with $E = J\rho$, Ohm's law for current density, and we do a final rearrangement to find $J = \frac{E}{\rho}$, and we have our answer $\boxed{D}$.

Free Response Questions

[10] **Problem 1.** *In the following parts, each item should be considered independently of the previous items.*

Consider a setup as shown in Fig 2.

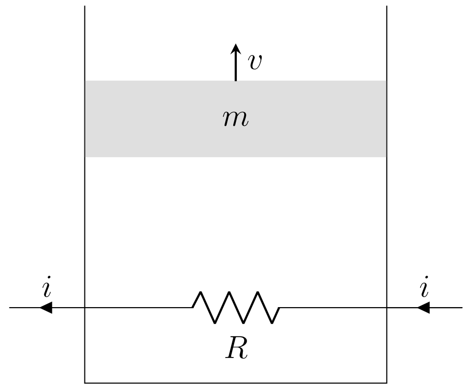


Figure 2: FRQ 1

A resistor with resistance R and current i runs through a thermally insulated container with an ideal gas inside, and a piston of mass m at the top. The piston then begins to move upwards with a speed v.

A) What should v be such that the energy of the system stays constant? Neglect changes in the energy of the gas.

B) The resistor is replaced with two resistors in parallel of resistance R, each with a current i running through them. Find v again.

C) Now assume as the gas has an internal energy proportional to T - that is,

$$E_{\text{int,gas}} = \alpha T$$

where α is a constant. Find the rate of change of temperature $\mathrm{d}T/\mathrm{d}t$ as a function of i, R, m, v, and any other constants.

Solution. A) In a time $\mathrm{d}t$, the piston has gained potential energy $\Delta U_g = mg\,\mathrm{d}y$. At the same time, the energy lost in the resistor is

$$\Delta E_c = -q\Delta V = -(i\,\mathrm{d}t)(iR) = -i^2R\,\mathrm{d}t$$

where the negative sign comes from the fact that energy is leaving the resistor as heat. Since the energy must stay constant, we must have

$$\begin{aligned} \Delta U_g + \Delta E_c &= 0 \\ mg\,\mathrm{d}y - i^2R\,\mathrm{d}t &= 0 \end{aligned}$$

Using $v = \mathrm{d}y/\mathrm{d}t$ we then have

$$v = \frac{i^2R}{mg}$$

B) The only difference in this case is that we must replace R with

$$R_{\text{eff}} = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{2}$$

and i with $i' = 2i$ (by the junction rule). Thus, we have

$$v = \frac{(2i)^2(R/2)}{mg} = \frac{2i^2R}{mg}$$

In other words, it will travel upwards twice as fast.

C) The process is largely the same as part A, except we have a contribution of $\Delta E_{\text{gas}} = \alpha\Delta T$.

$$\begin{aligned} \Delta U_g + \Delta E_c + \Delta E_{\text{gas}} &= 0 \\ mg\,\mathrm{d}y - i^2R\,\mathrm{d}t &= -\alpha\,\mathrm{d}T \\ \implies \frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{i^2R - mgv}{\alpha} \end{aligned}$$

[10] **Problem 2.** An initially uncharged capacitor is fully charged by an emf $\mathscr{E}$ in series with a resistor R (see Fig 3)

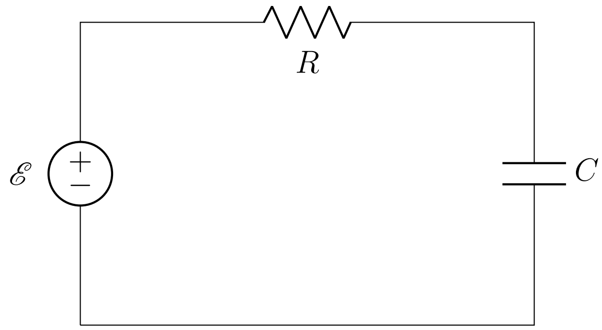


Figure 3: An RC circuit

A) After a very long time, the capacitor is charged to a charge q. What is the energy in the capacitor at this point?

B) Show the same result as above by directly integrating i^2R over the charging time.

C) At what time is the rate of heat dissipation in the resistor the same as the rate of energy storage in the capacitor.

Solution. A) The energy in a capacitor is $C(\Delta V)^2/2 = q\Delta V/2$. The change in potential is simply $\mathscr{E}$, as after a very long time the battery will have given up all of its potential. Thus

$$E = \frac{\mathscr{E}q}{2} = \frac{C\mathscr{E}^2}{2}$$

B) This is an RC circuit, for which the charge in the charge of the capacitor is given by

$$q = \mathscr{E}C(1 - e^{-t/\tau_{\mathrm{RC}}})$$

Using $i = \mathrm{d}q/\mathrm{d}t$ we have:

$$i = \frac{\mathscr{E}}{R}e^{-t/\tau_{\mathrm{RC}}}$$

Then,

$$\begin{aligned}\int_0^\infty i^2R\,\mathrm{d}t &= \frac{\mathscr{E}^2}{R}\int_0^\infty e^{-2t/RC}\,\mathrm{d}t \\ &= \frac{\mathscr{E}^2C}{2} \\ &= \frac{\mathscr{E}q}{2}\end{aligned}$$

Where the last line comes from $q = C\Delta V = C\mathscr{E}$.

C) The rate of energy storage is given by

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{q^2}{2C} \\ &= \frac{q}{C}\frac{\mathrm{d}q}{\mathrm{d}t} = \frac{q}{C}i\end{aligned}$$

The rate of dissipation of the resistor is i^2R. Setting the two equal gives

$$\begin{aligned}i^2R &= \frac{q}{C}i \\ iRC &= q \\ \mathscr{E}Ce^{-t/\tau_{\mathrm{RC}}} &= \mathscr{E}C(1-e^{-t/\tau_{\mathrm{RC}}}) \\ \implies t &= \tau_{\mathrm{RC}}\ln 2 = RC\ln 2\end{aligned}$$