Rotations Mock Exam

10 MCQ 2 FRQ - 60 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 9.8$ N/kg throughout this contest.
- Test under standard conditions, which means that you must complete the test in 60 minutes in one sitting.
- This test contains 10 Multiple Choice Questions and 2 Free Response Questions.
- Correct answers will be awarded the points shown; leaving an answer blank will be awarded zero points. The amount of partial credit is up to your discretion as the grader. Our recommendation is to be more strict than necessary.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a graphing calculator. Calculators may not be shared. Cell phones may not be used during the exam. You may not use tables, books, or formula collections.
- The number in **red** next to each question represents the amount of points the question is worth. There are a total of 40 points on this test.
- If you have any questions or clarifications, please contact us at tjhsstphysicsteam@gmail.com.

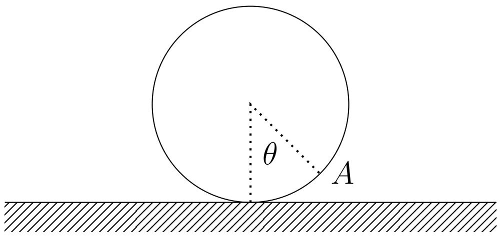
The creators of this exam are (in alphabetical order):

Aarush Deshpande, Eric Xie

Good luck and have fun! Turn the page to start the mock.

10 MCQs

[2] **Problem 1.** A wheel of radius R is rolling without slipping with an angular velocity ω. For a point A on the wheel at an angle θ with respect to the vertical, what is the magnitude of the velocity with respect to the ground?



(A) ωR

(B) $\omega R \sin(|\theta|/2)$

(C) $\sqrt{2}\omega R \sin(|\theta|/2)$

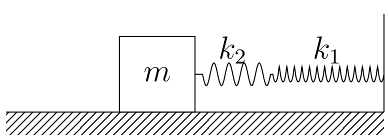
(D) $2\omega R \sin(|\theta|)$

(E) $2\omega R \sin(|\theta|/2)$

[2] **Problem 2.** A spring with a spring constant k_1 is attached to another spring with a spring constant k_2. k_2 is then attached to a mass, and the system begins to oscillate. The period of oscillation of the system is of the form,

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}}$$

What is k_{eff}?



(A) $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$

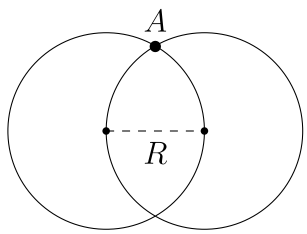
(B) $k_{\text{eff}} = \dfrac{k_1 + k_2}{2}$

(C) $k_{\text{eff}} = k_1 + k_2$

(D) $k_{\text{eff}} = \left(\dfrac{k_1}{k_2}\right)(k_1 - k_2)$

(E) $k_{\text{eff}} = \left(\dfrac{k_1}{k_2}\right)(k_1 + k_2)$

[2] **Problem 3.** Two *rings* of radius $R = 2$ m and mass 1 kg are attached a distance R apart, as shown in Figure 3. They rotate about a point A at the intersection of the two circles, with an angular velocity 4 rad/s. What is the angular momentum of the system, about an axis perpendicular to the paper and going through the point A?



(A) $16\text{kg} \cdot \text{m}^2/\text{s}$

(B) $32\text{kg} \cdot \text{m}^2/\text{s}$

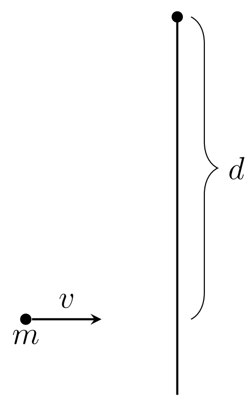
(C) $48\text{kg} \cdot \text{m}^2/\text{s}$

(D) $64\text{kg} \cdot \text{m}^2/\text{s}$

(E) $76\text{kg} \cdot \text{m}^2/\text{s}$

[2] **Problem 4.** Aarush holds a stick of mass m and length ℓ on one end, hanging down. Eric shoots a ball of mass m with velocity v, and it collides inelastically with the stick a distance $x = d$ from the top. What is the value of d for which Aarush doesn't feel a sting from the collision? Alternatively, at what value of d does the edge of the stick not move at the instant of collision if Aarush had not held the end in place? Ignore any effects from gravity.

The moment of inertia of a stick of length ℓ about an axis perpendicular to the plane going through its center of mass is $m\ell^2/12$.



(A) $\dfrac{\ell}{3}$

(B) $\dfrac{\ell}{2}$

(C) $\dfrac{2\ell}{3}$

(D) $\dfrac{3\ell}{4}$

(E) $\dfrac{5\ell}{6}$

[2] **Problem 5.** The following shapes roll without slipping down an inclined plane of angle θ, and length ℓ:

1. A disk of mass m and radius R.
2. A spherical shell of mass m and radius R.
3. A spherical shell of mass $2m$ and radius $R/2$.
4. A sphere of mass m and radius R
5. A sphere of mass $2m$ and radius R

Given the moments of inertia are $2mR^2/3$ for a spherical shell, $2mR^2/5$ for a solid sphere, and $mR^2/2$ for a disk, rank the times taken for each shape to reach the bottom of the inclined plane.

(A) $T_1 = T_2 = T_3 = T_4 = T_5$

(B) $T_1 < T_2 < T_3 < T_4 < T_5$

(C) $T_1 < T_2 = T_4 < T_3 = T_5$

(D) $T_2 = T_3 < T_1 < T_4 = T_5$

(E) $T_3 < T_2 < T_1 < T_4 < T_5$

(F) $T_3 < T_2 = T_5 < T_1 < T_4$

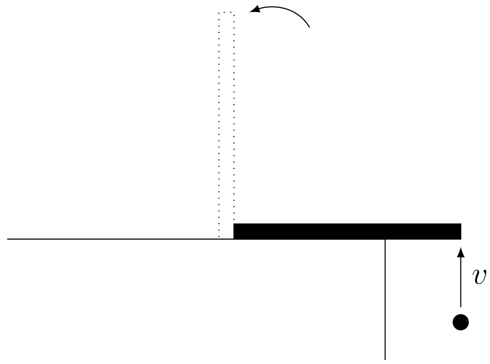
(G) $T_4 = T_5 < T_1 < T_2 = T_3$

(H) $T_4 < T_1 < T_5 < T_2 = T_3$

(I) $T_5 < T_3 < T_4 < T_1 < T_2$

(J) $T_5 < T_3 < T_1 < T_4 < T_2$

[2] **Problem 6.** A thin slab of mass m and length ℓ hangs over a ledge. There is a pivot on the left end of the slab that allows it to rotate about that point. A bullet of mass m and velocity v hits the bottom-right edge of the slab as shown and gets stuck inside it. The slab eventually comes to rest perfectly vertically. Given that the moment of inertia of a slab about its tip is $m\ell^2/3$, find v.



(A) $2\sqrt{gl}$

(B) $\sqrt{3g\ell}$

(C) $\sqrt{2g\ell}$

(D) $\sqrt{g\ell}$

(E) $\sqrt{\dfrac{8g\ell}{3}}$

[2] **Problem 7.** A thin disk of mass M and radius R is rotating about its central axis at an angular velocity ω_0. The disk has a very small hole made at a distance $R/3$ from its center. The hole is suddenly covered by a rod, and the rod stays at rest afterward. With what angular velocity does the disk start to rotate? Assume the disk is lying on a frictionless surface. The moment of inertia of a disk about its center is $MR^2/2$.

(A) $2\omega_0/3$

(B) $5\omega_0/8$

(C) $7\omega_0/13$

(D) $6\omega_0/7$

(E) $9\omega_0/11$

[2] **Problem 8.** A disk of radius 5 meters starts at rest at time $t = 0$. It undergoes uniform angular acceleration. At time $t = 3$, a point on the rim is now moving at 7.5 m/s. At time $t = 3$, what is the magnitude of the net acceleration that a particle 3 meters from the center experiences, assuming that the disk continues accelerating at time $t = 3$?

(A) 1.50 m/s^2

(B) 6.75 m/s^2

(C) 6.91 m/s^2

(D) 8.73 m/s^2

(E) 9.02 m/s^2

[2] **Problem 9.** Figure 1 shows a rotating skew rod. Two objects of mass 3 kg are attached to a rod a distance 4m away from the axis of rotation, such that the angular momentum L of the system about its center is at an angle $\alpha = 20°$ from the vertical, and the rod is spinning with an angular speed $\omega = 3\text{rad/s}$. Which of the following is the closest to the magnitude of $\mathbf{L}$?

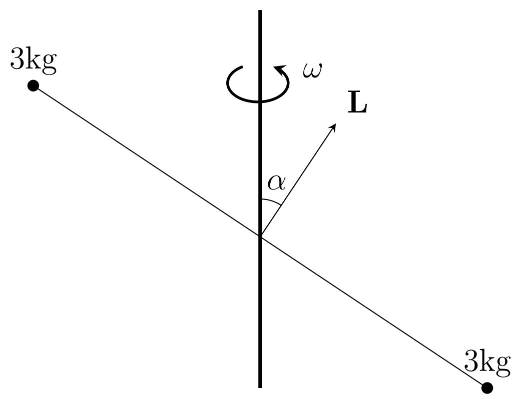


Figure 1: A Rotating Skew Rod

(A) $250 \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}}$

(B) $270 \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}}$

(C) $290 \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}}$

(D) $310 \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}}$

(E) $330 \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}}$

[2] **Problem 10.** One end of a stick of mass m and length ℓ is pivoted against a wall, and the other end rests on a frictionless floor (see Figure 2). Let F_l and F_R be the vertical forces on the left and right ends of the stick, respectively. Then:

(A) $F_L > F_R$

(B) $F_L < F_R$

(C) $F_L = F_R = mg/2$

(D) $F_L = F_R = mg$

(E) $F_L = F_R = mg \cos\theta$

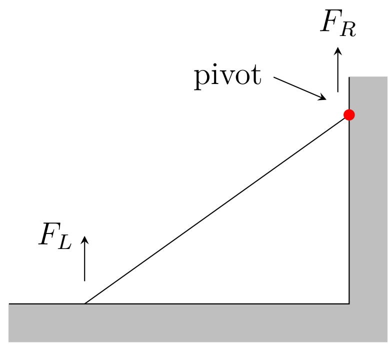


Figure 2: A Special Stick

Free Response Questions

[10] **Problem 1.** A circular disc is cut in half, to make a semicircle of mass m and radius R.

A) Find the moment of inertia of the new semicircle about the center of the original disk via integration.

B) The semicircle is then angled at a slight angle $\theta_0 \ll 1$ along its cut axis, and begins to oscillate with a period T. Find the distance from the axis of rotation to the center of mass of the semicircle in terms of T, m, R, and any fundamental constants. The moment of inertia of the semicircle about this axis is $mr^2/8$.

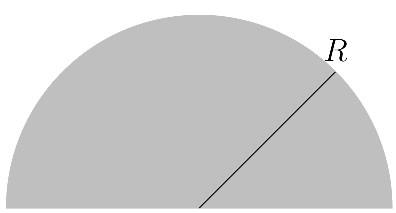


Figure 3: Part A

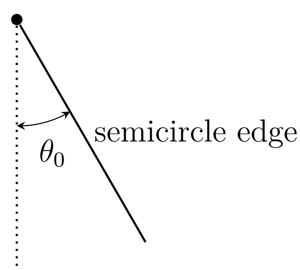
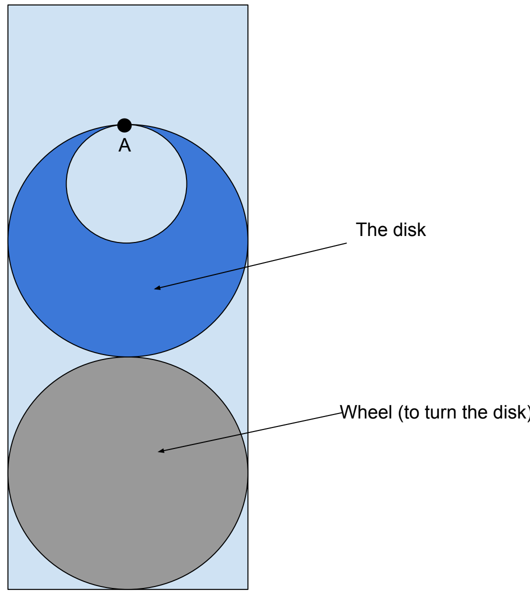


Figure 4: Part B, side view

[10] **Problem 2.** An initially uniform disk of radius R has a circular hole with radius R/2 cut out, as shown, where the center of the hole is a distance R/2 from the center of the disk. The mass of the remaining "disk" is M. The disk is placed in a container where it can only move in the vertical direction but can still rotate. At time $t = 0$, the disk is connected to a wheel that gives the disk an angular acceleration α counterclockwise.

The center of mass of this "disk" is given to be a distance R/6 from the center of the original disk. After some time T, the disk will start to "jump" up and lose contact the lower wheel momentarily because of the circular motion of the center of mass (it moves in a circle of radius R/6)



A) Find the time T where the disk begins to "jump" in terms of α, M, R, and any fundamental constants.

B) At some time t, find the angular momentum of the disk in terms of t, α, M, R, and any fundamental constants.

C) Using your answer for part b), find the angular momentum of the disk when it first loses contact with the wheel.